



What I Wish My Pastor Knew About ...
Mathematics and Beauty



<http://ministrytheorem.calvinseminary.edu>



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I will meditate upon the glorious splendor of Your majesty and on Your wondrous works.

-- Psalm 145:5

Other Worlds

In high school, I was a voracious reader. I loved passing my time reading science fiction and fantasy novels. It was a form of escapism for me. To imagine myself as a knight in Medieval times or a space explorer on an unknown planet was my favorite way of passing the time. So much so that if my studies failed to also feed my imagination in the same way, they often took second stage.

Then, in my sophomore year, I started encountering a genre of science fiction, written in particular by H.P. Lovecraft, in which appeared tales that saw the world and our everyday experiences as a façade behind which are hid deep and ancient mysteries where forces, often nihilistic, lurk and may someday make their appearance to the detriment of mankind. While this type of literature can rightfully be called dark and weird (which is the name the genre was given), it held a fascination for me for two reasons. First, much of the writing occurred during an era when research in physics was providing startling new insights into the nature of our physical world. Einstein's Special and General Relativity was beginning to give us a picture of large scale space and time as curved rather than flat, as previously thought, and quantum mechanics gave us a picture of the very small as exhibiting weird physical properties in which particles exist as energy packets, called quanta, which "move" in a discrete fashion. This view blurred our ability to make observable predictions of both their position and velocity. Such ideas as higher dimensions and non-Euclidean geometries provided fertile ground for imaginative stories. Often these stories contained descriptions of strange alien races who achieved bewildering advances in science and technology which we would begin to understand only when our mathematics and science one day reached a suitable maturity, perhaps millennia from now.

This leads to the second reason why these writings fascinated me so. They often found a way to convey a deep sense of wonder and mystery toward the unknown of our world, while portraying science and mathematics as a way for pulling back the many veils that hide these mysteries. As I was reading these stories, my geometry class was starting to hold more of my attention as I looked for such evidences of deeper mysteries in the geometric con-structions and deductions discussed in class. My math classes began to hold sway to my imagination in ways very similar to the science fiction tales that would enrapt my attention. Unfortunately, much of what my teachers were lecturing about generally appeared mundane compared with the fantastic mathematical forms conveyed in these stories. But in my senior year, as I was exploring possible colleges and universities to apply to, within their various course catalogs I saw in their math courses opportunities to see the topics of curved space and non-Euclidean geometry explored in ways that would perhaps hold for me the wonder that these stories had hinted were there to be found. From there, I made it my mission to focus my studies on mathematics and seek out the deeper mysteries possibly hidden with our reality.

Then I entered college and embraced Christianity and suddenly mathematics and wonder took on completely new meanings.

From wonder to beauty

"Where there exists wondrous proportion and primal equality ... "

- Saint Augustine, *On the Trinity* vi. 10

As is often the case, a college or university can be a transformative place for young minds. For myself, it was the place where my academic studies fed my imagination rather than being a distraction from the fantasies I sought to indulge. In particular, I found in my studies of mathematics the hints of the wondrous worlds that the stories I read in high school had suggested, a sense of something that transcended our everyday experience.

But very early on, in my freshman year, I encountered something else that also pointed to a transcendence beyond our world. I had made friends who demonstrated a life that showed a connection with this transcendence, one that spoke deeply of a divine creator who sought a deep relationship with men and women and initiated that through a historical act of incarnation. This faith that they shared with me opened my eyes to a true source of transcendence, a God who divinely created our reality and fills it with wonder and beauty and who provided the true way to knowing Him through the salvific work of his Son, as incarnated in Jesus Christ. Through this faith I saw a way to embrace more fully the possible wonders of this universe that I sought in my studies as ultimately and completely originating from a divine creator. Suddenly, our reality, and the potential realities mathematics spoke of, could all be seen illuminated by the same light, the light of Christ.

Now, what can be said of these wonders in mathematics that mesmerized me? Are they of the same sort that attract other people to the study of mathematics? At root to every subject in mathematics is both a sense of quantity and relations: geometry explores spatial relations, number theory explores natural number relations, analysis studies relations within continuous quantities, etc. Within each one of these subjects, mathematicians can find a venue to study such relations and find deep and elegant patterns. Seeking such hidden patterns serve as mysteries that mathematicians are often fond of finding and unlocking. Moreover, mathematicians further develop from these patterns sophisticated theories that expand the context in which these patterns can be found. These in turn provide a broader range of possibilities for applications within mathematics and, possibly, to other sciences. It is this process of pattern exploration, theory building and application which drives the development of the various subjects in mathematics. But within that process, mathematicians find points that can instill inspiration and wonder. Such points have certain features in common which individually or collectively can be said to portray a sense of

beauty. Here are some of those features.

- i. **Unexpected connections.** In a study of one or more mathematical subjects, two or more seemingly disparate objects or relations may suddenly be seen as shades of a single web of relations, providing a sense of unity within or across such subjects.
- ii. **Simplicity.** A theory that a mathematician seeks to develop aims to explain the truths underlying the patterns discovered through logical arguments grounded upon basic definitions, intuitive truths, and suitably basic constructions. Within such a framework, deep and unexpected connections are most intensely revealed within logical arguments in which the relations made can be communicated with the greatest simplicity, thus enabling both an ease in discerning their hidden truths and an ease in articulating and communicating such patterns to others.
- iii. **Openness to new possibilities and deeper connections.** Dwelling upon singular patterns and merely giving them a simple explanation is often insufficient and can result in simple dead ends. What can instill a deeper sense of inspiration to mathematicians is to develop within the theory a framework that not only explains those relations and patterns investigated, but provides a setting for surmising potential deeper relations and allows the language to open up and potentially express even possibly deeper connections previously unseen or guessed at.

Let us now move beyond these broad stroke descriptions of what mathematicians find beautiful and look at some concrete examples which demonstrate some of the features associated with beauty in mathematics.

Beauty, proof and symmetry

“Beauty is that which pleases when seen.”
– Saint Thomas Aquinas

If the reader would reflect for a moment upon a time when an experience produced a sense of wonder or beauty, I would suspect that immediately accompanying that experience was a further strong, positive, emotional experience which drew your very being to attend closer to that thing, perhaps to repeat that experience, or, perhaps, to see if further treasures of such experiences lie still beyond. This experience of beauty I would like to suggest is at play in the mathematician’s encounter with mathematics, one which reflects a sense of the unveiling of mysteries found in the encounter of deeper connections within the forms and patterns being discerned and deciphered. The type of beauty that is found within a mathematician’s world is one which produces wonder and instills desire for further exploration, there is a magnetic or gravitational attraction that pulls upon the mathematician’s attention, focusing their full being. This notion of aesthetics in mathematics can be initially encapsulated in the most concise expression of beauty, due to Thomas Aquinas, as “that which pleases when seen”: the moment of encounter with this object of beauty brings pleasure to all the ways it is viewed, grasped, and sensed.

To give a reader a concrete case study in mathematical beauty, we will concentrate on a subject that often provides the easiest gateway to mathematics through the senses, namely geometry. This most ancient of rigorous mathematics can provide an entryway to encountering beauty in two particular ways: rigorous proof and symmetry. For this paper, we shall spend more time with the latter, but, for the moment, it should be noted that rigorous proof is an important source of aesthetic encounters for the mathematician. As evidence of this, consider the following observation of Sir Bertrand Russell:

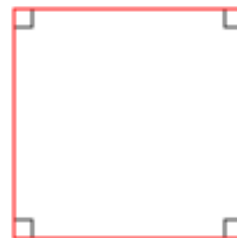
At the age of eleven, I began Euclid, with my brother as my tutor. This was one of the great events of my life, as dazzling as first love. I

had not imagined that there was anything so delicious in the world.

Or the even more poetic and succinct is the following observation by Edna St. Vincent Millay: “Euclid alone has looked on Beauty bare.”

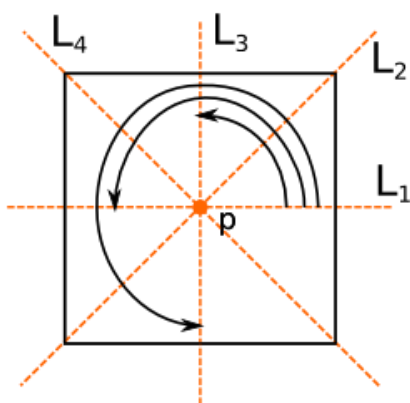
What is it in the structure of Euclid’s Elements that evokes such responses of wonder in those that take it upon themselves to study this work? Euclid, in putting together the Elements, produced a paradigm for organizing a body of knowledge. Beginning with basic key geometric definitions and self-evident postulates which govern the way these geometric notions relate, the theory of plane geometry is developed from the basic to the more sophisticated through further definitions and constructions, with deeper relations disclosed in propositions. These in turn are established through rigorous proofs that unveil how preexisting relations can be weaved together through logic in order to arrive at the desired result. Euclid’s approach to conveying mathematical knowledge in his Elements has been considered the ideal approach to organizing any subject of mathematics and persists to today. It thus provides the means to find beauty in all parts of mathematics in ways that resonate with what Russell and St. Vincent Millay found in Euclid’s Elements.

Now let’s consider a certain approach to geometry in which the notion of beauty presents itself in a more common sense fashion, namely through the notion of symmetry. The idea behind symmetry is rather simple. Consider the following square.



Symmetries of this square can be imagined in two different ways. First, imagine rotating the square counterclockwise through a fixed angle. A rotation

is a symmetry if the square after rotation “looks the same,” such that the vertices and edges have moved to the positions of other vertices and edges). If you can imagine this, you should conclude that the rotations through 0, 90, 180, and 270 degrees give all the useful rotational symmetries (any symmetric rotation of 360 degrees or more can be viewed as one of these four). Below is a square¹ with some of these rotations rendered.

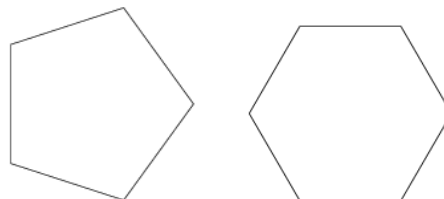


The second type of symmetry is a reflection, as illustrated in this same diagram. Fixing your attention upon any one of the four lines L1, L2, L3, and L4 crossing through the center point of the square, imagine spinning the square in space around that fixed line until it lies back in the plane. This gives a reflective symmetry. It is a geometric result that all planar symmetries of the square that fix the center point are either one of the four rotations or one of the four reflections, totaling eight possible planar symmetries. Furthermore, a given symmetry can be related to another symmetry through some third symmetry by a method of composition in which performing one symmetry then applying a second symmetry will result in a third symmetry. For example, rotating the square counterclockwise 90 degrees then reflecting through a horizontal line L1 gives a symmetry that is identical to reflecting the square around the diagonal axis L2. Collectively these symmetries, together with this method of composition, form an example of a structure

¹ From [Cornel Math Explorers Club](#)

called a *group*.

Now an analysis of symmetries can be carried out for any geometric shape, not just the square. For example, any polygon in the plane, like the regular pentagon and hexagon below,² has symmetries.

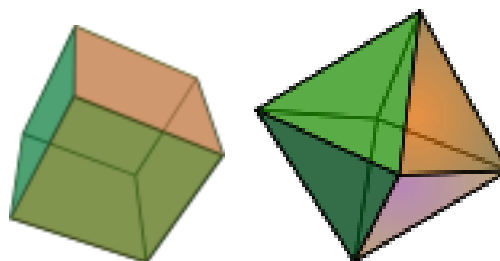


The regular pentagon has five rotations and five reflections in its group of symmetries, the regular hexagon has six rotations and six reflections in its group of symmetries, and so on.

If we consider now geometric objects in three spatial dimensions, the analog of regular polygons are the *regular polyhedra*, also known as the *Platonic solids*. In contrast to regular polygons, in which their number is infinite, there are exactly five Platonic solids³: the tetrahedron



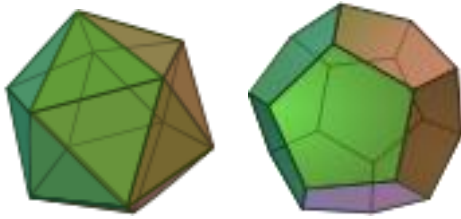
the cube and octahedron



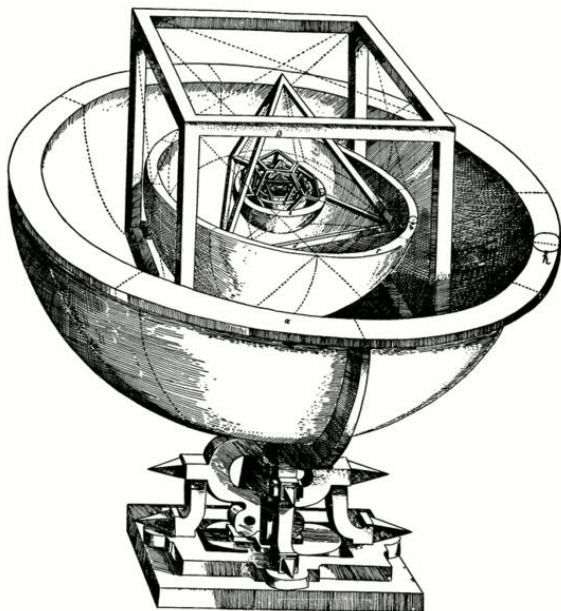
and the dodecahedron and icosahedron.

² http://en.wikipedia.org/wiki/Regular_polygon

³ http://en.wikipedia.org/wiki/Regular_polyhedron



Each of these solids carries a group of spatial symmetries as well. Recall that for planar objects, the rotations and reflections were about lines, but for these solid objects the rotations and reflections are through planes. As a source of inspired beauty, many mathematicians, philosophers, and scientists, such as Euclid, Plato and Kepler, have found in the Platonic solids such deep aesthetic pleasure as to seek ways to make them building blocks of the universe. For example, Euclid's *Elements* concludes with characterizations and a complete classification of the Platonic solids and Kepler, in his *Mysterium Cosmographicum*, gave a model rendering the solar system using nested Platonic solids⁴.



It wasn't until the 19th century that

mathematicians fully developed the theory of symmetries and their groups. It is from this theory that geometric objects were studied and characterized through their groups of symmetries.

At the start of the 20th century, the revolutions in physics – special and general relativity theory and quantum physics – found in the mathematical theory of symmetry the means to model the quantitative properties and relations in the newly understood nature of space and time of the very large or the very small. For example, in Einstein's theory of relativity, his principle of invariance asserts that the same experiment conducted at two different points in space and time will have essentially the same outcome once the appropriate space-time symmetry is taken into account. In quantum physics, the most fundamental of particles possess internal symmetries that individually characterize them as the particles they are. Furthermore, the way these particles interact with each other possess a wealth of symmetries that both characterize their relationships between them and provide the means to locate them experimentally in, for example, particle accelerators. In the current regime of theoretical physics research and exploration, the endeavor to find a Grand Unified Theory – an endeavor to produce a model of fundamental particles that accounts for all the forces of nature – has led physicists to seek further extensions of the theory of symmetries of space-time in order to expand our current accounts of particle physics to also include gravity and relativity theory. For example, *superstring theory* incorporates a ginned up version of symmetry known as *supersymmetry*. Thus, symmetry illustrates all three types of beauty mentioned earlier: unexpected relationships of symmetries into groups, simplicity of the visible geometry of the symmetric relationships, and new possibilities when applied to the physical world.

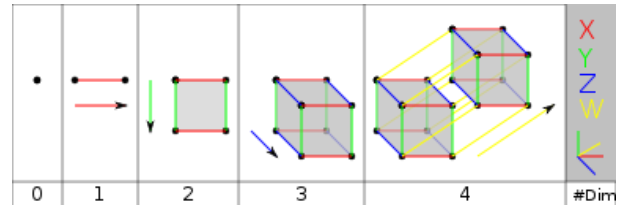
⁴ http://en.wikipedia.org/wiki/Johannes_Kepler

Dimensions

Another major advance in mathematics in the 19th century that also played a pivotal role in the advances of 20th century physics was the development of geometry in higher dimensions. Ever since Descartes created analytic geometry in the 17th century, Euclid's geometry in the *Elements* could be synthesized with algebra in such a way that mathematicians found rather quickly from within their algebraic equations in many variables lurked intricate geometric objects that required more than three spatial dimensions to describe. Moreover, the calculus as created by Newton and Leibniz could be generalized and applied to such geometric objects to enable a deeper understanding of their nature. All of this culminated in the 19th century with the work of Bernhard Riemann who developed a general theory of geometry that unified the algebraic and analytic features explored since the 17th century. This view of geometry required mathematicians to unshackle their senses in order to comprehend those features of geometric objects that resided in four, or five, or even higher dimensions. This breaking free of our senses in order to comprehend a reality that completely transcends our three-dimensional world requires thinking by analogy to translate our experiences to those of a creature who inhabits such a world of higher dimensions. This inspired authors like Edwin Abbot to pen a book like *Flatland* which describes two dimensional creatures which live in a planar world and tells the tale of one such creature (a square) who is paid a visit by a three dimensional creature (a sphere). What unfolds is the attempts of the sphere to explain his nature to square, who can only experience a world of two dimensions. One of Abbot's aims in this novel is to give the reader a window into the then recent exploration by mathematicians into the nature of higher dimensions, which illustrates the idea of transcendence in both its scientific and religious sense.

To see how analogy can give insight into higher

dimensions, consider the following sequence⁵ of geometric figures.



The progression from left to right portrays the notion of cube in the appropriate dimension:

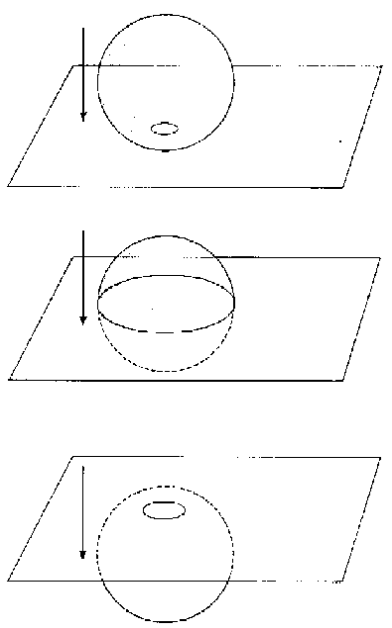
- a point is a 0 dimensional cube;
- a line segment is a 1 dimensional cube formed by dragging the point one unit to the right along a one dimensional axial direction;
- a square is a 2 dimensional cube formed by dragging the line segment one unit along a second direction perpendicular to the first axial direction;
- a cube is a 3 dimensional cube formed by dragging the square segment one unit along a third axial direction perpendicular to both the first and second axial directions;
- a hypercube (or tesseract), then, is a 4 dimensional cube formed by dragging the cube segment one unit along a hypothetical fourth axial direction perpendicular to each of the first, second and third axial directions.

Since we cannot experience such a fourth axial direction, we are left with only a conceptual description to how we may form a hypercube based on how lower dimensional cubes are formed. Properties of the hypercube are then extracted through this analogy.

This approach of using analogy to visualize and analyze higher dimensional objects is one of the main ways mathematicians can translate such objects to ones in our realm of experience for study. This process is necessary as visualizing higher dimensional objects in their actual form is extremely difficult. By imagining how such objects can be constructed from lower dimensional objects, as described above, we may

⁵ <http://en.wikipedia.org/wiki/Tesseract>

surmise the properties of objects in dimensions 4, 5, and higher in an inductive way by beginning with our experience with developing objects from dimensions 1, 2, and 3. This is the method of analogy that is so beautifully described by Abbott in Flatland, particularly in the Flatland character Square’s encounter with the 3 dimensional Sphere. As Sphere entered into Flatland, Square perceived a dot which became a circle whose radius grew until it reached Sphere’s radial length, and then shrunk back down until the circle reached a point and then disappeared, as illustrated in the next figure.⁶

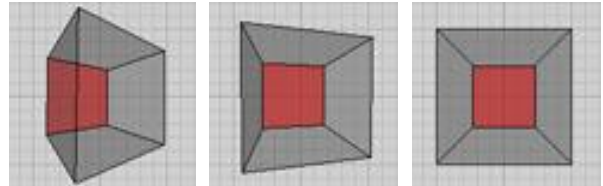


By analogy, one way to imagine a hypersphere in 4 dimensions is by its appearance as it passes through our physical 3 dimensional space. Initially we would see a point and then, like a balloon, we see a small sphere that inflates until it expands to a sphere of radial length equal to that of the hypersphere and then deflates back down to a point.

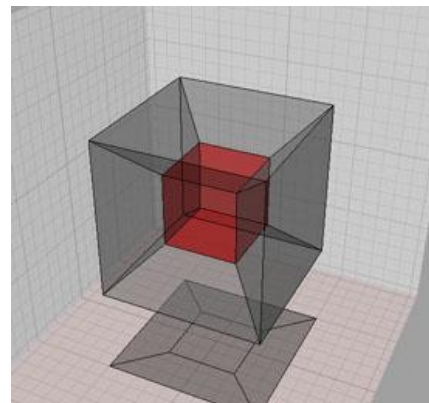
Another way to visualize the hypercube through

⁶ From [Dwight E Vincent](#)

analogy is to consider the cube through the following sequence of perspectives⁷.



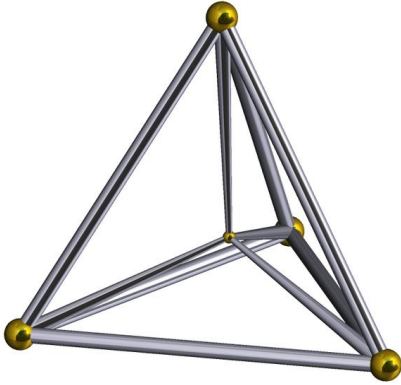
Here the cube is to be viewed by focusing on the red back face as the viewer comes from the side of the cube, moving until facing directly the back face through the front face. This last perspective of the cube (on the right) can also be viewed in a two-dimensional fashion, as an outer square and an inner square with nearby corners connected by line segments. This way of viewing a cube two dimensionally is often called a *projection* or *shadow* of the cube. By analogy, we may consider a similar perspective of a hypercube: our three dimensional view of the cube that focuses on the back face through the front face translates to a four dimensional view of the hypercube that focuses on the “back cube” through the “front cube”. The resulting projection from four space to three space translates “inside/outside square” for the two dimensional projection of a three dimensional cube to “inside/outside cube” for the three dimensional projection of a four-dimensional cube and appears as follows.



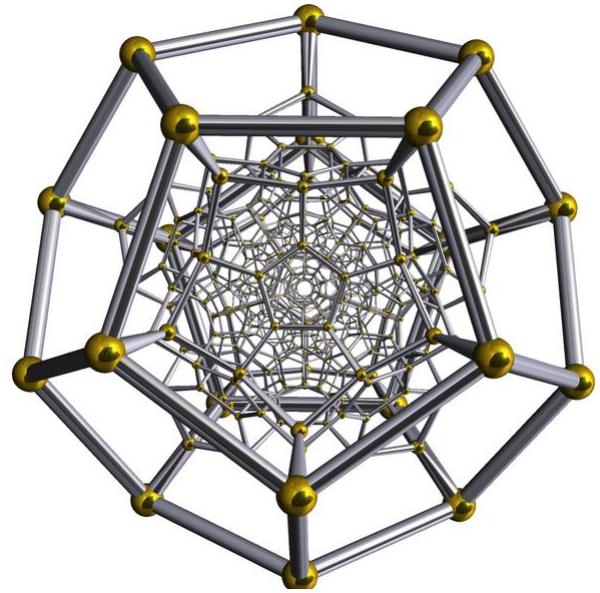
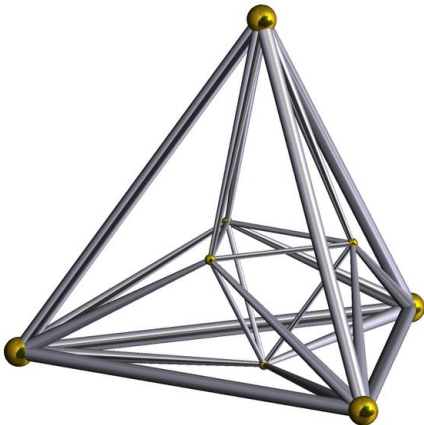
In a similar way, the other Platonic solids possess

⁷ From [Drew Olbrich](#)

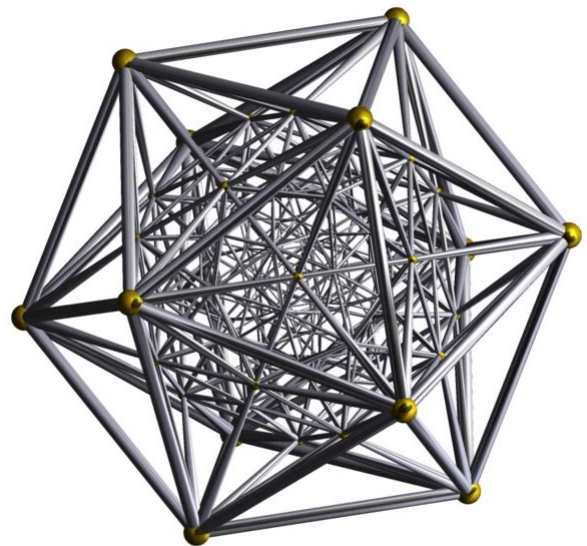
four-dimensional analogs. For example, below are the projections⁸ for the hyper-tetrahedron,



the hyper-octahedron,



and the hyper-dodecahedron.



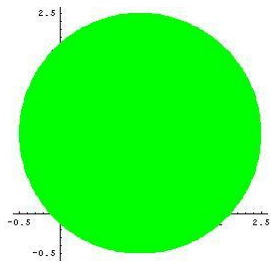
Notice the remarkable symmetry and intricacy in the hyper-dodecahedron and the hyper-dodecahedron. The simple visual symmetry of these images is a great example of beauty in mathematics.

the hyper-dodecahedron,

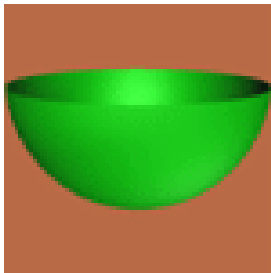
⁸ http://en.wikipedia.org/wiki/Regular_polytope

Now, higher dimensional geometry, as noted

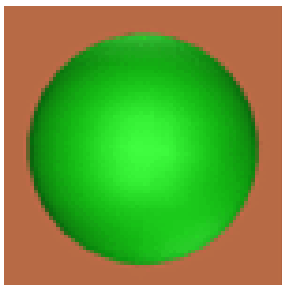
before, has found its way into scientists' efforts to understand the universe in the 20th century through the developments of general relativity theory and quantum physics. One recent way such higher dimensions have entered physical theories is in cosmology and the effort to describe the large-scale structure of the universe. To get a feel for how these higher dimensions are contemplated, consider another exercise in analogy. Consider the disk⁹.



This is a two dimensional object. If we consider this disk as viewed edge-on in space and push the center downward, we get a bowl.



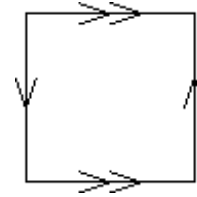
Stretching the rim of the bowl to touch a point above the bowl forms a sphere.



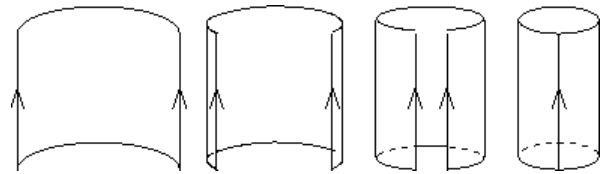
Now consider the square with the sides oriented as

⁹ From [The Geometry Center](#) at University of Illinois Urbana-Champaign

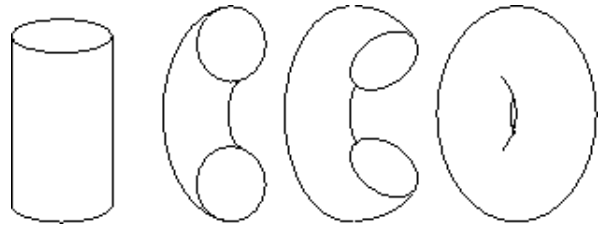
follows¹⁰



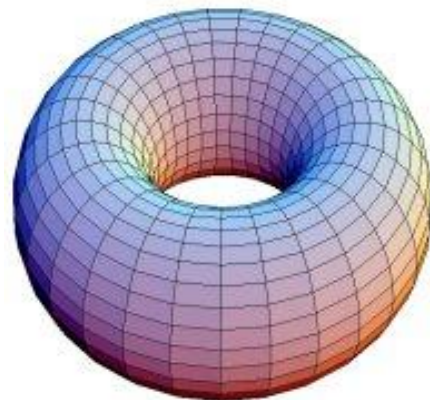
Connecting the opposing left/right sides



gives a cylinder. Gluing the top and bottom together



gives an inner tube shape, called a *torus*¹¹



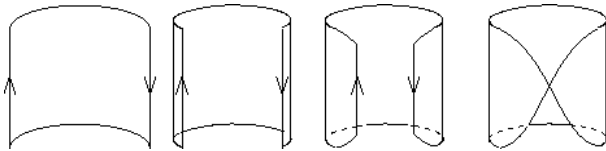
Thus by simply gluing the edges of a square

¹⁰ From [The Geometry Center](#) at University of Illinois Urbana-Champaign

¹¹ From [Oliver Knill](#)

together, we can get a shape that looks completely different. Notice, from the per-spective of the square, that travelling to one edge transports the traveler back to the opposite edge, which accounts for the two perpendicular circular directions on the torus. Imagine living as an ant on the square - every time you departed the left edge you'd appear on the right edge; the same thing happens seamlessly on the torus.

Consider next a similar square in which only the left/right sides have arrows oppositely directed. The process of identifying those sides can be seen as follows



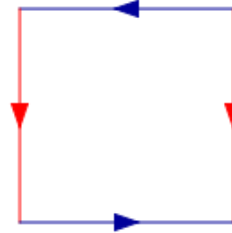
The resulting glued object is called a *Möbius band*. Such an object possesses the feature of being “unoriented” in that, in contrast with a cylinder, it fails to have a distinct inside and outside i.e. it is *one sided*. This can be seen through M.C. Escher’s picture of ants crawling along the Möbius band¹²



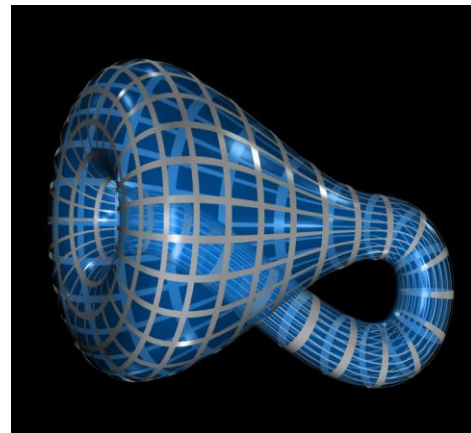
Finally, consider the square with edges oriented as

¹² From http://artchive.com/artchive/E/escher/escher_red_ants.jpg.html

follows¹³



Gluing opposite edges in the way that aligns the arrows yields an unoriented surface known as the Klein bottle¹⁴



The unorientability of this surface prevents it from having both an inside and an outside: an ant can crawl from the inside to the outside without reaching an edge. The picture is deceptive, to view it in three dimensions, as above, requires the neck of the bottle to pierce the body. Four spatial dimensions are required to give a proper depiction. This is the sort of hidden reality that excited me as a student; simply by matching up edges on a square can yield a bizarre shape that can't be portrayed in 3 dimensions.

Now, there are three features of these three surfaces, the sphere, the torus and the Klein bottle, that we can make note of:

- i. *Locally two dimensional*. Because we formed

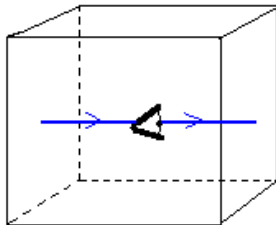
¹³ http://en.wikipedia.org/wiki/Klein_bottle

¹⁴ From [Thomas Banchoff and Jeff Beall at Brown University](#)

them by gluing the edges of a flat square, the surface looks flat and two dimensional when focusing on any point close up .

- ii. *Closed.* While the square has an edge or boundary, after gluing the edges the boundary disappears making it edgeless or closed.
- iii. *Embedded in higher dimensions.* Even though these surfaces are two dimensional up close, being closed forces them to live three or even four spatial dimensions.

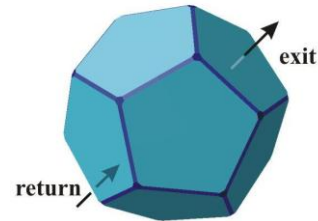
A similar view can be given to the description of our universe. Cosmologists develop models of the cosmos based upon general relativity and supported by astronomical observations. Among them there are some models that are geometrically closed. We know from our own experience that the world is locally three-dimensional. What are the possible geometric descriptions of such a closed three-dimensional structure? By analogy, instead of starting with a square and selecting rules for gluing the outer edge, start with a cube¹⁵



Imagining the interior to be our universe, we may consider opposite faces glued according to variations on the gluing rules we contemplated for the square. Performing this gluing for all three pairs of opposing faces gives a closed, locally three-dimensional object which, because of gluing all opposing faces, requires embedding in more than three spatial dimensions to properly exist. Note, as with the torus, that viewing beyond a face brings one's visual field back into the cube through the opposite side, right behind the viewer (you could see the back of your own head). It should be noted, that other polyhedra can be

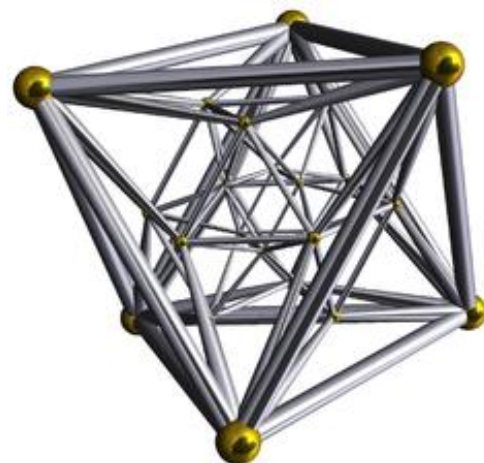
¹⁵ <http://www.etsu.edu/physics/etsuobs/starprty/12/0598bg/section6.htm>

considered when forming models of the universe. For example, Jeff Weeks¹⁶ has proposed, based on some astronomical observations, that gluing opposing faces of dodecahedron¹⁷



gives a good closed model of our universe. This geometric object is called a *Siefert-Weber manifold*.

For a mathematician, the exploration of higher dimensions need not end at four dimensions. For example, in our discussion of Platonic solids, we identified the only five that exist in three-dimensional space. In four-dimensional space, we indicated there is a hyper version of each of the five of the Platonic solids. Are there others? The definition of hyper-Platonic solids does not necessarily exclude other possibilities and, in fact, there is one more, called the *24-cell*, whose projection into three dimensional space is displayed here:¹⁸



Now the definition of Platonic solids and hyper-

¹⁶ Jeff Weeks, *The Shape of Space*
¹⁷ From l'Observatoire de Paris
¹⁸ <http://en.wikipedia.org/wiki/24-cell>

Platonic solids can be generalized to five, six, seven and beyond to the notion of *regular polytope*. Can a similar classification be given to such objects in such higher dimensions, or do things become overly complex? Well, the notions of hyper-tetrahedron, hyper-cube, and hyper-octahedron persist easily to every dimension. The amazing thing is that *in each dimension of 5 or higher, the only regular polytopes are the appropriate analogs of the hyper-tetrahedron, hyper-cube, and hyper-octahedron*. Here we have a wondrous example of the treasures that can be found within the mathematician's imagination. In the expectation that higher dimensions implies higher complexity, which in general is true, the high order symmetry of regular polytopes restricts its possibilities to just the most basic types. Unfortunately, or fortunately, geometric objects in higher dimensions generally can take on a variety of complex and exotic features for which any specific assertions that can be declared by a mathematician regarding them often require a list presuppositions in order to get a firm grasp upon their nature. However, in the case of Platonic solids, the use of analogy and symmetry as a means to discern these geometric objects leads not only to the type of simplicity and unexpected connections that gives a breathtaking beauty to our understanding of higher dimensions, but also provides a way to initiating the search for deeper connections as it relates to more general geometric objects in any dimensions by first considering them as a suitably general form of polytope.

The Splendor of Creation: Beauty and the Glory of God

"From the greatness and beauty of created things proportionally the maker of them is seen."
 – Wisdom 13:5

By taking geometric objects and their symmetries as the source of examples of beauty in mathematics, my aim is to give emphasis on aspects of mathematics whose presence is also strongly felt in science, particularly physics. I hope this gives the reader a sense of how

mathematics instills a sense of awe in mathematicians and scientists as they explore the deep inner workings of physical space. But, moreover, one can retrieve from such explorations a further sense that physical space is not required to be the way it is. Rather, from the viewpoint of mathematics, there is a wealth of possibilities for how space can be woven together to give a geometry for the fabric of the cosmos. From such a vantage point, one can easily be drawn in to see the universe as a creation – a creation intended to be appreciated and marveled at by participants with whom such a Creator would wish to have a relationship. Such participants may be rapturously led to see from the equations they are contemplating and which disclose the geometric designs encountered, the impress of a divine author at the root of its entire existence. Herein is true beauty found: to see the presence of the Creator revealed in the designs spoken in the relations eloquently articulated through the equations of the physicist or mathematician. Furthermore, it is in the expressions found in the mathematician's world that the colors of creation's possibilities can be discerned to be among those on the Creator's palette, perhaps as seen before the brush has even touched the canvas.

In conclusion, among the ways that beauty finds its presence within a mathematical discourse include the unexpected connections revealed within the physical makeup of reality, the pleasing encounter with and fruitful productivity from symmetric relations, and the contemplation of transcendent realities within higher dimensions surmised through the power of analogy. Each of these can elicit awe from the mathematician, the scientist, the pastor and the parishioner alike as they examine the nature of space in its geometric forms. That we may contemplate the ways reality both *is* and *could be* is a source of great mystery. If one is willing to step back to take it all in, it can inspire a sense of awe and a consideration of the possibility of a divine author to all that there is – perhaps leading the one contemplating to respond in the most profound fashion: Glory!

Further Reading

Beauty in mathematics and science

- *Divine Proportion* by H. E. Huntley
- *Mathematics and the Aesthetic: A New Approach to an Ancient Affinity* eds. N. Sinclair, D. Primm and W. Higginson
- *Truth and Beauty: Aesthetic Motivations in Science* by S. Chandrasekhar

Symmetry

- *Why Beauty is Truth: A History of Symmetry* by Ian Stewart

Geometry, polyhedra, and higher dimensions

- *Euler's Gem* by David S. Richeson
- *Shape of Space* by Jeff Weeks
- *Flatland: a Romance of Many Dimensions* by Edwin Abbott Abbott
- *Flatland the movie* available at <http://www.flatlandthemovie.com/>

Beauty and theology

- *The Beauty of the Infinite* by David Bentley Hart
- *Faith and Beauty: A Theological Aesthetic* by Edward Farley
- *Spirit and Beauty* by Patrick Sherry